FD-2862

BCA (Part-II) Examination, 2022

Paper - I

Calculus and Differential Equations

Time : Three Hours] [*Maximum Marks* : 80

[*Minimum Pass Marks* : 27

Note : Answer any **two** parts from each question. All questions carry equal marks. Simple/Scientific calculator is allowed.

Unit-I

1. (*a*) Prove that $\lim_{x \to 3} \left(x^2 + 2x\right) = 15$.

(*b*) State and prove the Mostest theorem.

(*c*) Test for differentiability the function f given by

$$f(x) \begin{cases} x^m \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Also find the value of m when $f'(x)$ is continuous at $x = 0$.

Unit-II

2. (*a*) Find the derivative of the function $\log_{10} x + \log_x 10$ with respect to x.

(*b*) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, then prove that

$$x^2 y_{n+2} + (2n+1)\, x y_{n+1} + 2n^2 y_n = 0$$

(*c*) Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = a^2$ at the points where a common ordinate cuts them. Show that if θ be the greatest inclination of these tangents, then

$$\tan\theta = \frac{(a-b)}{2\sqrt{ab}}.$$

Unit-III

3. (*a*) Evaluate

$$\int \frac{x \tan^{-1} x^2}{1+x^4} dx$$

(*b*) Evaluate

$$\int \frac{dx}{\sin(x-a)\sin(x-b)}$$

(*c*) Evaluate

$$\int \log\left(1+x^2\right)dx$$

Unit-IV

4. (*a*) Show that

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx, \text{ if}$$

$$f(2a-x) = f(x), \text{ and}$$

$$\int_0^{2a} f(x)\,dx = 0 \quad \text{if } f(2a-x) = -f(x)$$

(*b*) Find the value of $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$.

(*c*) Show that

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}}\log\left(1+\sqrt{2}\right)$$

Unit-V

5. (*a*) Show that $Ax^2 + By^2 = 1$ is the solution of

$$x\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=y\frac{dy}{dx}$$

(*b*) Solve the differential equation $(1-x^2)\ (1-y)\ dx = xy\ (1+y)\ dy$

(*c*) Solve

$$(x+y)\ (dx-dy) = dx + dy$$